

GENERAL STUDY ON TYPE OF EDGE SETS GRAPH

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ABSTRACT

This paper of sets loads two ways the first one is path given by logicians. They have analyzed theory in great detail and have formulated axioms for the subject each of their axioms expresses a property of sets that mathematicians commonly accepts. The other goes up, onto the high lands of mathematics itself where these concepts are indispensable in almost all of pure mathematics as it is today here in this paper we introduce the ideas of set theory and establish the basic terminology and notation. The new thing which we shall give in this paper is the concept of 'set of graphs'. The set of graphs will consist only simple graphs.

1.1 INTRODUCTION

1.1.1 Set :

A set is a well defined collection of objects each of which satisfies a certain property such that it enable us to decide as to whether the given objects belongs to that collection or not. Commonly we shall use capital letter A, B, to denote the sets and small letters a, b, c..... to denote the objects of elements belonging to these sets.

If an objects x belong to a set A we denote it by the notation $x \in A$.

If x does not belong to A we express it by the notation $x \notin A$.

We say that A is a subset of B if every element of A is also an element of B and are we express by $A \subset B$.

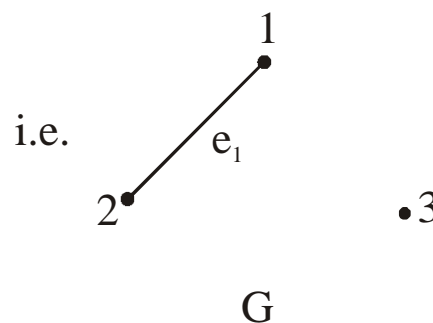
If $A=B$ it is g true that both $A \subset B$ and $B \subset A$. If $A \subset B$ and $B \subset A$. If $A \subset B$ and A is different from B we say that A is proper subset of B and we write $A \subset B$.

1.2 SET OF GRAPHS :

Let us assume a simple graph $G(V,E)$, where V is the set of vertices and E is the set of edges. In our course of study, we shall concern ourself only on the set of edges E and throughout the work it is called as the set of graphs. In a simple graph $G(V,E)$, if there are n edges then the set of graphs will contain 2^n sub graphs with respect to the edge set i.e. E.

Now let us take some examples to clear the concept of the set of graphs.

Example : Let us consider a simple graph $G(V,E)$, where $V=\{1,2,3\}$ and the edge set $E=\{e_1\}$.



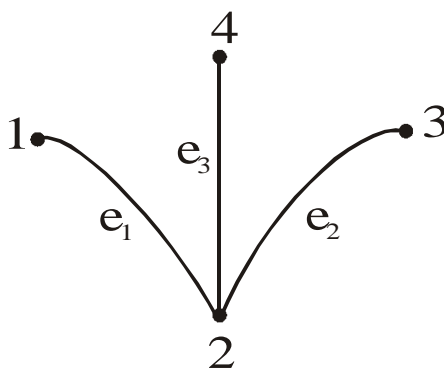
As there exists only one edge in the graph G, hence $\exists 2^1$ elements in the set of graphs. thus the subset of E will be ϕ & $\{e_1\}$.

Thus the elements in the set of graphs are $\{\phi, \{e_1\}\}$.

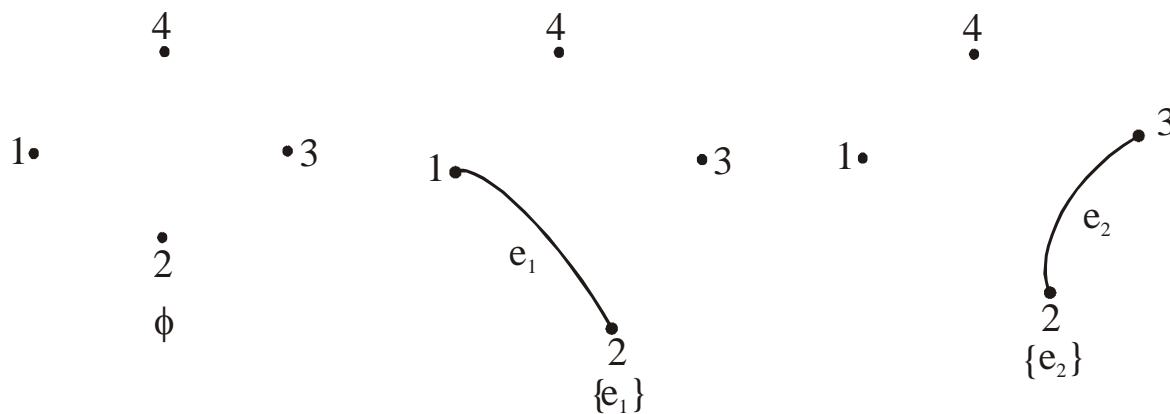


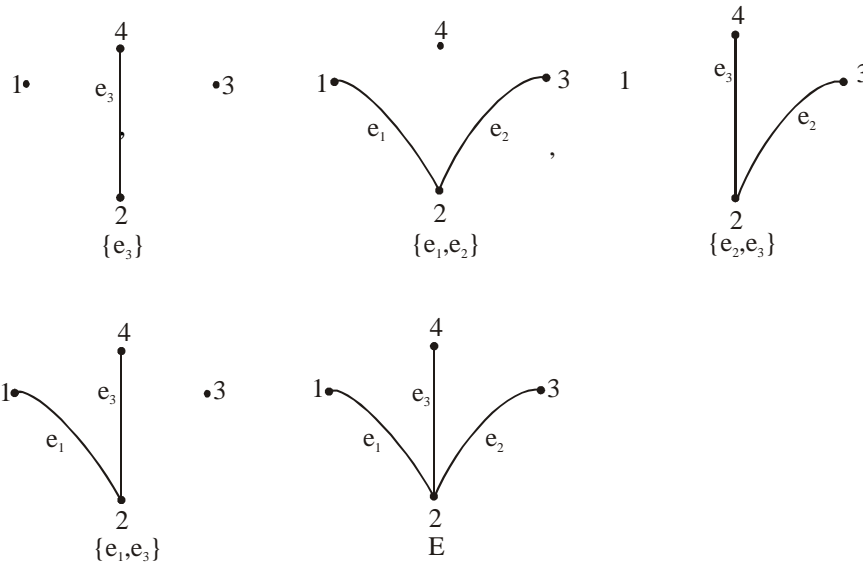
For the better understanding of the above given concept, let us take two more example.

Example : Let us take a simple graph $G(V,E)$, where $V = \{1,2,3,4\}$ and the edge set $E = \{e_1, e_2, e_3\}$.



The given graph G is defined over three edges. Thus the set of graphs will contain 2^3 i.e.. 8 sub graphs.
 Set of graphs = $\{\phi, E, \{e_1\}, \{e_2\}, \{e_3\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_2, e_3\}\}$.
 This set of graphs can be represented as follows:





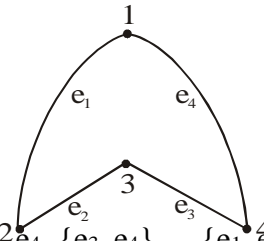
Here we can observe that null set and the edge set E always be member of the set of graphs.

Example : Similarly we will find the set of graphs on the edge set of four edges.

Let us assume an edge set $E = \{e_1, e_2, e_3, e_4\}$ on the simple graph $G(V, E)$.

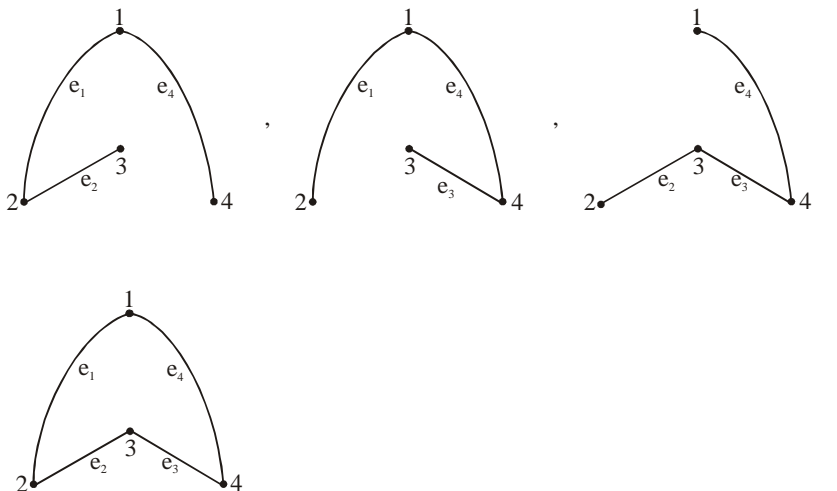
As it is given that there exist four edges. Then by the above defined deaf. there will exist 2^4 sub graphs *i.e.* 16 sub graphs.

Thus, define these elements of the set of graph *i.e.*



$\{\phi, E, \{e_1\}, \{e_2\}, \{e_3\}, \{e_4\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_3, e_4\}, \{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_3, e_4\}, \{e_2, e_3, e_4\}$, is the set of graphs.

So the sub graphs are following:



Now, we have come to a conclusion that the set of graphs can be defined on any simple graph like a any set of elements.

1.3 TYPES OF EDGE SET:

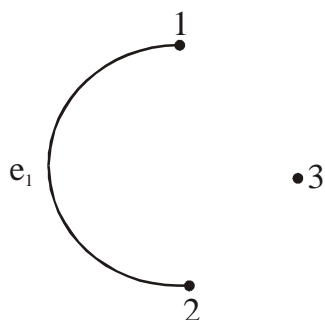
The set of edges have same types of sets as in case of simple sets of elements. Different types of edge set can expressed as follows:

1.3.1 Singleton Set:

An edge set will be singleton set if it contains only one element i.e. only on edge.

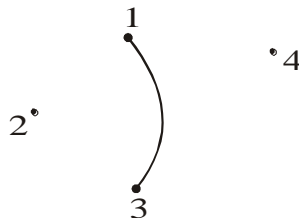
Example : let us consider a simple graph containing a single edge. Let the graph is $g(V,E)$, where $n(V)=3$ simple edge. Let the graph is $G(V,E)$, where $n(V)=3$ and $\eta(E) = 1$ i.e. $V = \{1,2,3\}$ & $E = \{ e_1\}$.

As the graph G has only one edge, so it is called singleton set irrespective of the fact that it contains there vertices.



One more example of singleton set is as follows :-

Example:- Let the graph is $G(V,E)$, where no. of vertices are 4 and no. of edges is 1. i.e. $V=\{1,2,3,4\}$, $E =\{e_1\}$ because the graph G has only one edge, so it is called singleton set. The graph is as follows:-



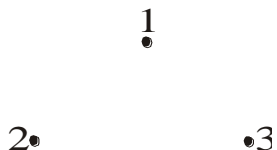
1.3.2 Null Set:

Any edge set is said to be the null set if it cantons no edge i.e. empty edge set of a simple graph $G(V,E)$. where $\eta(E)=\phi$.

Example : let us consider an example for better understanding.

let the graph $G(V,E)$ is a simple graph, where $\eta(V)=3$ i.e., $V\{1,2,3\}$ and $\eta(E) =\phi$ i.e. $E\{\}$

Thus the edge set of the graph G is empty set i.e.

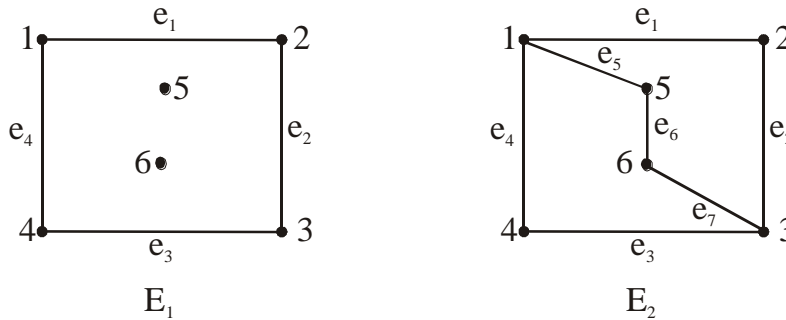


edge set is a null set.

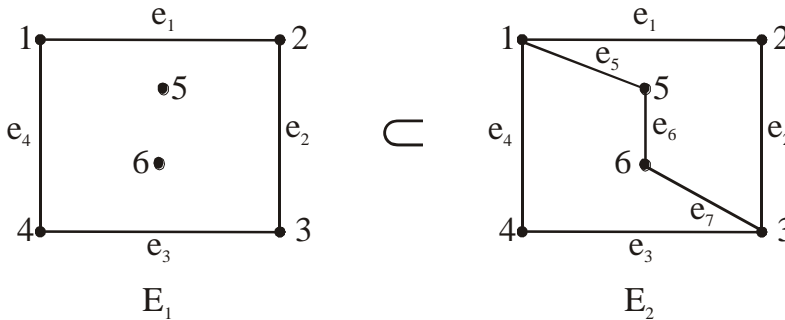
1.3.3 Subset or Superset: Two edge set E_1 , and E_2 , are such that every edge of E_1 , belongs to the edge set E_2 , then E_1 is called the subset of E_2 , & E_2 , is called the superset of E_1 ,

Example: Let us assume two edge set E_1 and E_2 , such that $t_1 = \{e_1, e_2, e_3, e_4\}$ and the edge set $E_2 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$
 Let present the edge set E_1 & E_2 in graphical form

and



As we can see that the edge set E_2 is containing all the edges of E_1 is the subset of the edge set E_1 . It can be shown as:-



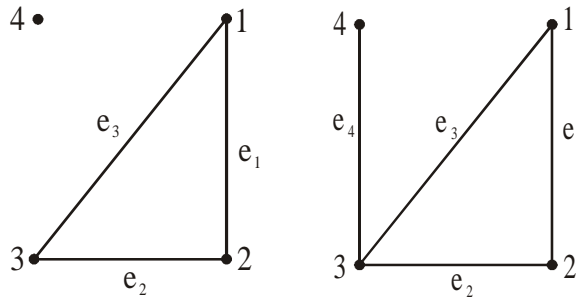
On the same time, we can say E_2 is the superset of E_1 as it contains all the edges of E_1 i.e. $E_2 \supset E_1$.

1.3.4 Proper Subset: The edge set E_1 is said to be proper subset of E_2 if every edge of E_1 is contained in the set E_2 but there exist at least one edge of E_2 which does not lie in the set E_1 .

i.e. $E_1 \subset E_2$ but $E_1 \neq E_2$

Example : let us consider two edge set E_1 and E_2 such that

$E_1 = \{e_1, e_2, e_3\}$ i.e.

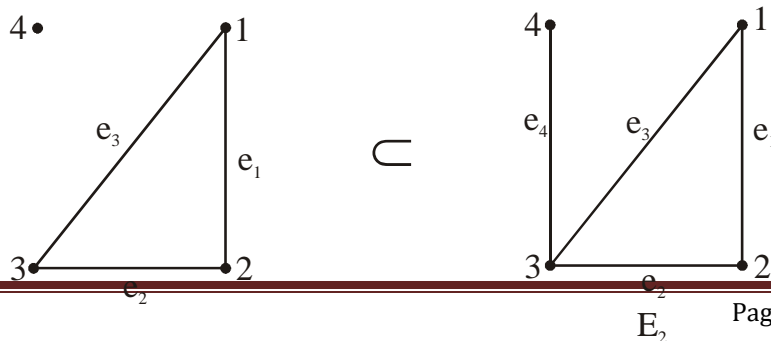


and the another edge set E_2

can be represented as following:-

On analyzing, both the edge set, we can conclude that the edge set E_2 contains all the edges of the edge set E_1 and one more edge is there in E_2 which is not in the edge set E_1 . i.e. $E_1 \neq E_2$

Thus E_1 is the proper subset of E_2 .



1.3.5 Equivalent Sets :

Two edge set E_1 and E_2 are said to be equivalent if the number of edges in E_1 and E_2 are same whether the edges are same or not i.e. the no. of edges should be same.

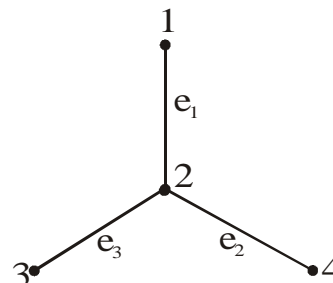
OR

two edge set are said to be equivalent if the number of edges are same irrespective of the fact that whether edges are same or not.

i.e. $n(E_1) = n(E_2)$

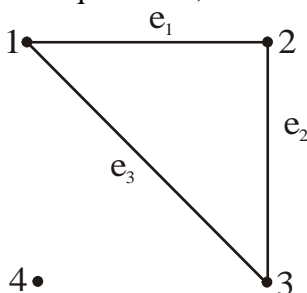
Example: let us assume two edge set E_1 and E_2 such that:

$E_1 = \{e_1, e_2, e_3\}$ i.e.



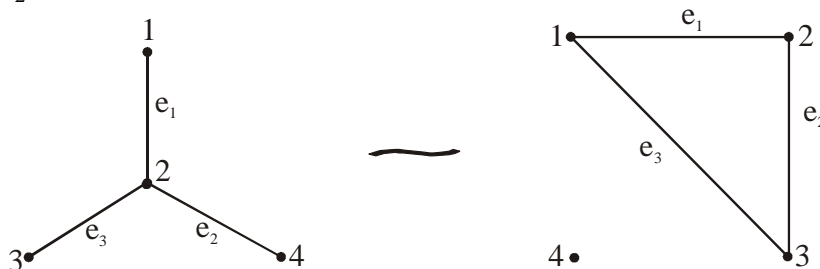
and the another edge set E_2 can be represented as

Now the edge set E_1 is not equal to E_2 , as the edges of E_1 and E_2 are not same.



But no. of edges are same, it mean there exist there edges in both the edge set.

Hence $E_1 \sim E_2$

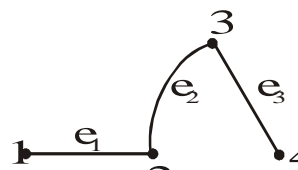


1.3.6 Power Set :

If an edge set e is defined on n number of edges, then the power set of E will contain 2^n elements i.e. all the subsets of E and denoted by $P(E)$.

let's verify it with the help of example.

Example: let us assume an edge set E on the given graph G .

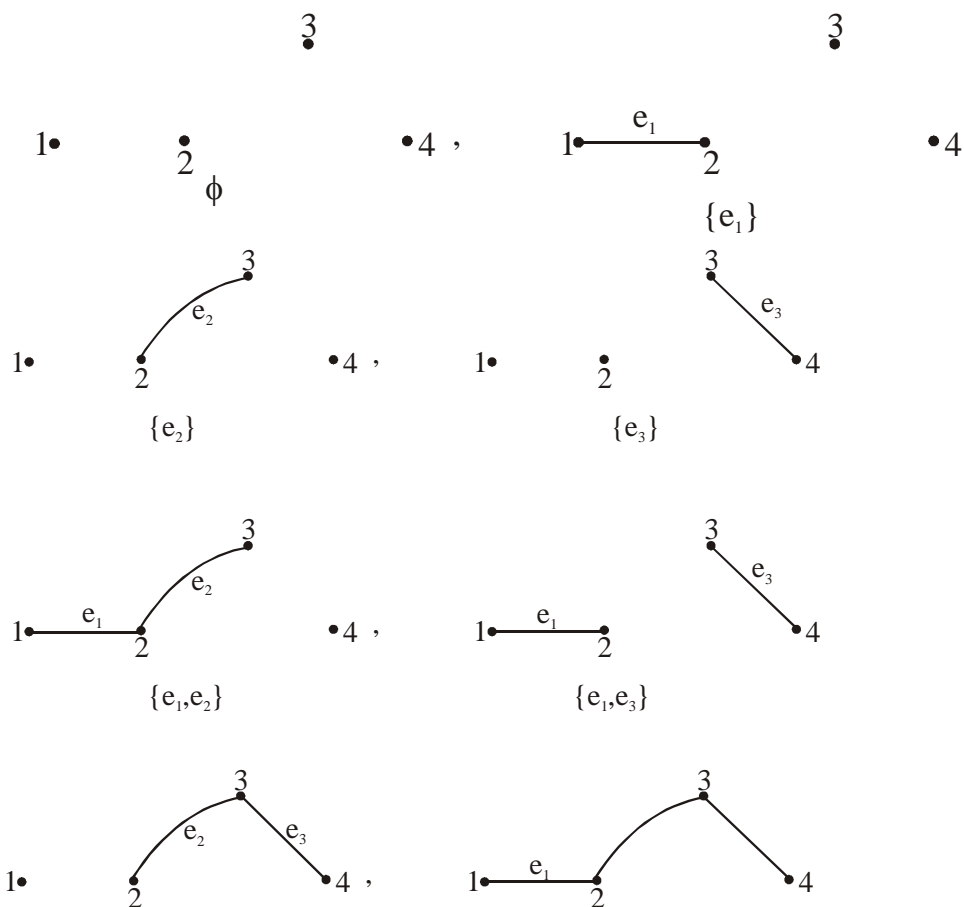


As the given edge set E is containing three edges, the $P(E)$ will contain 2^3 edge set i.e. 8 edge subset.

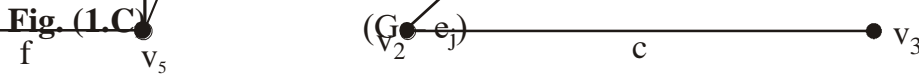
Thus the power set of the edge set E will contain the elements:-

$$P(\phi) = \{ \phi, \{e_1\}, \{e_2\}, \{e_3\}, \{e_1, e_2\}, \{e_1, e_3\}, \{e_2, e_3\}, E \}$$

These edge subset can be depicted as following:



Hence it can conclude that the edge set has similar type of sets as in the set of elements.



1.4 CONCLUSION

This survey paper on set-valuations of graphs which includes various type of set indexers, we hope that it will pave the way for any researcher for studying the topic. The conjectures and open problems identified in various sections appear would be quite interesting, for further investigation. In this perspective, the authors wish a general study

on set-valuations of graphs would be a long term goal. So far we have discussed the properties and characteristics of a new types of graphs called edge-set graphs derived from the edge sets of given graphs. Another important area that offer much for further investigations is the study on the edges of the edge-set graphs and set-graphs of given graphs. More studies are possible on the comparison of the set-graphs and edge-set graphs derived from the edge sets of different graphs. All these facts highlight the wide scope for further research in this area.

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